

GENERAL CONCEPTS ABOUT SOLID BODY ASTRODYNAMICS

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ABSTRACT: The paper presents the theoretical foundation of the dynamics of the solid body in gravitational movement in the outer space. It is known that the movement of solid bodies in space is subjects both the laws of Newton's classical mechanics and those of Kepler's celestial mechanics. Thus, the necessary conditions whom a solid body must fulfil to be able to leave the terrestrial surface and be launched into space are established. Also, the parameters that define the trajectories that solid bodies can have in the gravitational field are presented.

KEYWORDS: astrodynamics, orbital mechanics, escape velocity, orbital speed

1. INTRODUCTION TO ASTRODYNAMICS

Orbital mechanics or astrodynamics is the application of ballistics and celestial mechanics to the practical problems concerning the motion of solid body in space, as the rockets, artificial satellites and spacecraft [1-3]. The movement of these bodies is usually calculated using laws taken from classical mechanics: Newton's law of motion and universal gravitation law. Orbital mechanics is a basic discipline that allows the design and control of the space mission. It treats more the orbital dynamics of solid bodies under the influence of gravity, including spacecraft as well as natural astronomical bodies such as star systems, planets, natural satellites and comets. The orbital mechanics focuses on spacecraft trajectories, including orbital maneuvers, changes in orbit, and interplanetary transfers, and is used by mission planners to predict the results of propulsive maneuvers. General relativity is a more accurate theory than Newton's laws for calculating orbits, and sometimes is required for greater computational accuracy or in high gravity situations (such as the orbits of planets closer to the Sun) [4].

2. PRINCIPLES AND TECHNIQUES APPLIED IN ASTRODYNAMICS

The following principles and techniques contained in a compendium of applicative rules are useful for approximated situations by classical mechanics in accordance with standard assumptions formulated in astrodynamics. The most specific and frequently case for analysed is that of a satellite that orbits a planet, but these rules could apply to other situations, such as orbits of a small solid bodies around a star such as the Sun. Thus [5]:

- Kepler's laws on the planetary motion:
 - The orbits are elliptical, with the heavier body placed in one of the ellipse focus points. Particularly: Orbit has a circular trajectory, where the circle is a special case of the ellipse, and the planet is located in the system center.
 - A line drawn from the planet to the satellite measures equal areas in equal time, regardless of the measured orbit portion.
 - The square of the orbital period of a satellite is proportional to the cube of the mean distance from the planet.
- Without applying an external force, such as the firing force emitted by a rocket engine, the period and shape of the satellite's orbit will not change.
- A satellite that gravities on a low orbit or on the low part of an elliptical orbit, moves faster than the planet's reference surface relative to another satellite that revolves over a larger orbit or on the higher part of an elliptical orbit, due to higher gravitational field of the satellite located closer to the planet.
- If the external force is applied only in one point from the satellite orbit, the satellite will return to the same point on each subsequent orbit, although its trajectories will always change. Consequently, one cannot move from one circular orbit to another with a single brief application of a thrust pulse.
- If to one satellite placed on a circular orbit was applied a thrust pulse in a direction opposite to motion, then its trajectory changes into an elliptical orbit, and the satellite will descend and reach the lowest orbital point called periapsis, located to 1800 away from the firing point, after which it will ascend back to the initial impulse point. If the satellite receives a traction impulse in the direction of motion, then its trajectory changes into an elliptical

orbit, but the satellite will climb to the highest point of the trajectory, called apoapsis, located to 1800 away from the firing point, after which it will descend back to the initial impulse point.

3. LAWS OF ASTRODYNAMICS

The fundamental laws of astrodynamics, as was outlined in paragraph 1, are the Newton's universal gravity law and the motion laws of solid bodies enunciated by Newton, and the mathematical solution of these laws is based exclusively on differential calculus [1, 2].

The first equation of motion is:

$$F = ma \quad (1)$$

where: F is force [N], m is mass [kg] and a is acceleration [m/s²]

The Earth creates a gravitational field $E(r)$ given by:

$$E(r) = \frac{Gm_E}{r^2} \vec{r} \quad (2)$$

where: $m_E = 5.972 \times 10^{24}$ kg is a constant value and represents the mass of the Earth, $G = 6.674 \times 10^{-11}$ N·m²/kg² is the gravitational constant, r is the distance from Earth to solid body (satellite) [m] that gravitates around the Earth, and \vec{r} is the vector of this distance (the derivative by the first order of this distance is velocity) [m/s].

The gravitational force acting on the solid body in motion is equal to:

$$F_{cp} = -\frac{Gm_E m \vec{r}}{r^2} = -\frac{Gm_E m r}{r^3} \quad (3)$$

This force acts inwardly, i.e. to the Earth. For this reason it is defined as a centripetal force acting on the solid body. Since the product Gm_E is a constant, it can be defined:

$$\mu = Gm_E = 3.986 \times 10^{14} \text{ Nm}^2/\text{kg}$$

which is known in the literature as Kepler's constant. By entering the above value in relation (3), it is obtain:

$$F_{cp} = -\frac{\mu m r}{r^3} \quad (4)$$

In order to maintain the solid body on a well-defined trajectory, simultaneously acts on it an F_{cf} centrifugal force, equal and opposite to the F_{cp} centripetal force. The balance of these two forces makes the solid body move on a certain trajectory without falling into the center of mass 0. The dynamics of these two forces is illustrated in Figure 1.

If points A and B are very close, then:

$$\frac{\overline{AB}}{\overline{OA}} = \frac{v dt}{r} \approx \frac{dv}{v} \quad (5)$$

Knowing:

$$\frac{dv}{dt} = \frac{v^2}{r} \quad (6)$$

the centrifugal force of the solid body is given by:

$$F_{cf} = m \frac{dv}{dt} = m \frac{v^2}{r} = \frac{mv^2 \vec{r}}{r} = \frac{mv^2 r}{r^2} \quad (7)$$

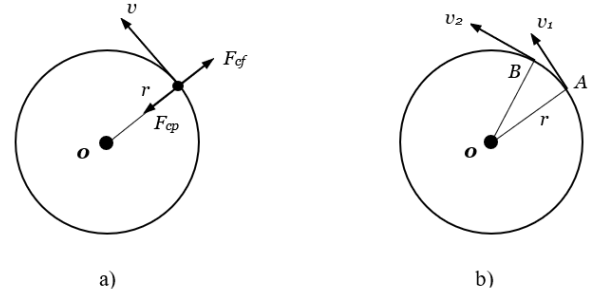


Figure 1.

- a) forces acting on the solid body
b) vectors velocity at two different moments

Since the sum of the forces acting on the solid body must be zero, from equality of relations (4) and (7) results in:

$$\frac{\mu m}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{\mu}{r}} \quad (8)$$

A direct consequence of this equation is that the velocity of the solid body is inversely proportional to its orbital altitude r . The lower the orbit of the solid body, the faster it travels in space. To determine the length of time T needed for a solid body to go through a particular orbit, must calculate the circumference S of that orbit:

$$T = \frac{S}{v} = \frac{2\pi r}{\sqrt{\frac{\mu}{r}}} = 2\pi r \sqrt{\frac{r}{\mu}} \quad (9)$$

It can be noticed that the higher the orbital altitude r of a solid body, the longer its orbital period. The orbit of a solid (satellite) body can be graded according to the corresponding orbital altitude and period, as shown in Table 1.

Table 1.

Orbit	Orbital altitude /km/	Orbital period T
Low Earth Orbit	160 – 2 000	87 – 127 min
Medium Earth Orbit	2 000 – 35 786	127 min – 24 hr
Geostationary Earth Orbit	35 786	23 hr 56 min 4.1 sec

Any orbit and trajectory outside the terrestrial atmosphere is, in principle, reversible, i.e. in the space-time coordinate system, the time parameter is reversed. The displacement velocities are also reversed, while the accelerations remain the same, including those due to the bursts of the rocket engines. Thus, if a burst of the rocket engine creates an impulse in the direction of the velocity, according to the reversibility principle, it opposes to velocity.

Of course, in the case of rocket engines explosions, there is no complete reversibility of the events, although in both cases the same differential calculation of the velocity and the same mass ratio applies.

Some standard assumptions have been adopted in astrodynamics, which refer to non-interference from the outside solid bodies, negligible mass for one of the bodies, and the neglect of insignificant forces such as solar wind, atmospheric drag, etc. More accurate calculations can be made without these simplifying hypotheses, but they are more complicated. Increased accuracy often does not make enough difference in the calculation to be useful.

Kepler's laws on planetary motion can be derived from Newton's laws when it is assumed that only the gravitational force of the central solid body acts on the solid (satellite) body which is in orbital motion. When a thrust force or propulsion force is present, Newton's laws continue to apply, but Kepler's laws are invalidated and are not applicable. When the thrust force or propulsion force ceases, the resulting orbit will be different from the original one, but it will again be described by Kepler's laws. The three laws on our solar system are:

1. The orbit of a smaller solid body relative to another larger solid body is always an ellipse with the center of mass located in the larger solid body, in one of the two focus point of the ellipse. In other words, the orbit of any planet is an ellipse with the sun placed in one of the outbreaks.
2. The orbit of the smaller solid body measures equal areas in an equal amount of time. That is, a line joining a planet and the sun sweeping equal areas in equal time intervals.
3. The squares of the orbital periods of the smaller solid body are directly proportional to the 3rd power of the semi-major axis of the orbits multiplied by a constant.

4. ESCAPE VELOCITY

Escape velocity v_e is the slowest velocity a solid body has to have for escape the gravitational attraction of a certain planet or other heavier solid body. If the solid body slows at a minimal orbital velocity, it will hit the planet; if it accelerates beyond the maximum escape velocity, it will be permanently removed from the planet [6].

The formula for calculated an escape velocity is easily derived as follows. The specific energy (energy per unit mass), called ε , of any spacecraft is composed of two components, the specific potential

energy ε_p and the specific kinetic energy ε_k . The specific potential energy of the solid body associated with a planet of mass M , having the gravitational acceleration G and located at distance r , is given by:

$$\varepsilon_p = -\frac{GM}{r} \quad (10)$$

while specific kinetic energy to same solid body is given by:

$$\varepsilon_k = \frac{v_e^2}{2} \quad (11)$$

By applying the law of energy conservation:

$$\varepsilon = \varepsilon_k + \varepsilon_p \quad (12)$$

and so the total specific orbital energy, is obtained:

$$\varepsilon = \frac{v_e^2}{2} - \frac{GM}{r} \quad (13)$$

which does not depend on the distance r from the center of the planet, i.e. the central solid body, to the space vehicle in question. Therefore, the space vehicle can reach infinite r only if the escape velocity is not negative. In the case where $\varepsilon = 0$, the escape velocity must fulfill the condition:

$$v_e \geq \sqrt{\frac{2GM}{r}} \quad (14)$$

For information, the escape velocity on the Earth's surface is about 11 km/s, but this is insufficient to send the solid body (space vehicle) to an infinite distance due to the gravitational attraction of the Sun. To escape from the Solar System from a location situated at a certain distance from the Sun, equal to the Sun-Earth distance but not close to Earth, a velocity of about 42 km/s is required. If a space vehicle is launched in the displacement direction of the Earth, which is in its orbital motion around the Sun, acceleration due to its own propulsion system requires much lower values.

5. ORBITAL VELOCITY

According to standard simplification assumptions, the orbital velocity v_o of a solid body moving along an elliptical orbit, can be calculated with equation [4, 6]:

$$v_o = \pm \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a} \right)} \quad (15)$$

where: μ is the standard gravitational parameter [Nm^2/kg], r is the distance between the orbital solid bodies [m] and a is the length of the major semi-axis [m].

In the case of a hyperbolic trajectory, the orbital velocity of the solid body may have a positive or negative value depending on the conventional sense of movement of the solid body on its orbit.

6. ORBITAL SPECIFIC ENERGY

In the case of standard simplifying assumptions, the orbital specific energy ε of a solid body gravitating on an elliptical orbit is negative, and the orbital equation for orbital energy conservation corresponding to this orbit, can take the form [3]:

$$\frac{v_0^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} = \varepsilon < 0 \quad (16)$$

where: v is the velocity of the orbiting solid body [m/s], r is the distance between the orbiting solid body and the gravity center of the central body [m], a is the length of the major semi-axis [m], μ is the standard gravitational parameter [Nm²/kg].

For a given major semi-axis, the specific orbital energy is independent of the magnitude of the eccentricity. In some particular cases:

- the mean value of the specific potential energy ε_p is equal to 2ε
- the mean value of the distance $r-l$ is equal to $a-l$
- the mean value of the specific kinetic energy ε_k is equal to $-\varepsilon$.

7. CALCULATING TRAJECTORIES. KEPLER'S EQUATION

A way of calculating the trajectories of solid bodies, viewed from the perspective of a historical approach, is based exclusively on Kepler's equation:

$$\begin{cases} x + \frac{1}{60}x^3 + \frac{1}{1400}x^5 + \frac{1}{25200}x^7 + \frac{43}{17248000}x^9 + \frac{1213}{7207200000}x^{11} \dots & |x = (6M_a)^{\frac{1}{3}} \quad \varepsilon = 1 \\ \frac{1}{1-\varepsilon}M_a - \frac{\varepsilon}{(1-\varepsilon)^4} \frac{M_a^3}{3!} + \frac{(9\varepsilon^2+\varepsilon)}{(1-\varepsilon)^7} \frac{M_a^5}{5!} - \frac{(225\varepsilon^3+54\varepsilon^2+\varepsilon)}{(1-\varepsilon)^{10}} \frac{M_a^7}{7!} + \dots & \varepsilon \neq 1 \end{cases} \quad (19)$$

Alternatively, Kepler's equation can be solved and numerically. Initially, an arbitrary value is assigned to it and the time-of-flight is calculated, then are given more consecutive values E_a , as much as needed to bring the calculated time-of-flight closer to the desired value until the required precision is obtained. Typically, Newton's method is used to achieve relatively rapid convergence.

The main difficulty with this approach is that it may take too long to achieve a satisfactory convergence of extreme elliptical orbits. For almost-parabolic orbits, eccentricity e is close to value 1 . Entering this value ($e = 1$) into the formula for mean anomaly ($E_a - e \cdot \sin E_a$) shows that two almost equal values are subtracted and the accuracy suffers. For almost-circular orbits it is difficult to determine the periapsis, because the truly circular orbits have no periapsis at all. In addition, the equation was derived from the hypothesis of an elliptical orbit, and so it is not valid for parabolic or hyperbolic orbits.

$$M_a = E_a - e \cdot \sin E_a \quad (17)$$

where: M_a is the mean anomaly [m], E_a is the eccentric anomaly [m], and e is the eccentricity [m].

With Kepler's formula it is possible to determine the time-of-flight of the solid body to reach the value 0 of the periapsis angle, which is the true anomaly. The calculation procedure comprises two steps [7]:

1. Determination of eccentric anomaly E_a from true anomaly 0
2. Determination the time-of-flight at time t against eccentric anomaly E_a

Finding the eccentric anomaly at a given time (the inverse problem) is more difficult. Kepler's equation is transcendental in E_a , which means it cannot be solved algebraically. Determination of the eccentric anomaly E_a from the Kepler's equation can be done by analytical way, by inversion.

A solution of Kepler's equation, valid for all real values of e is [8]:

$$E_a = \begin{cases} \sum_{n=1}^{\infty} \frac{M_a^n}{n!} \lim_{\theta \rightarrow 0} \left(\frac{d^{n-1}}{d\theta^{n-1}} \left(\frac{\theta}{\sqrt{\theta - \sin(\theta)}} \right)^n \right), & \varepsilon = 1 \\ \sum_{n=1}^{\infty} \frac{M_a^n}{n!} \lim_{\theta \rightarrow 0} \left(\frac{d^{n-1}}{d\theta^{n-1}} \left(\frac{\theta}{\theta - \varepsilon \cdot \sin(\theta)} \right)^n \right), & \varepsilon \neq 1 \end{cases} \quad (18)$$

By developing these relationships, it is obtain:

8. GRAVITY AND THE OBERTH' EFFECT

In a known and controlled gravitational field, a spacecraft located in the atmosphere of a planet, can leaves it and go in a different direction at a different velocity [7]. This is useful to accelerate or slow a spacecraft, avoiding so the transport of a surplus fuel.

This maneuver can be approximated by an elastic collision at great distances, although the change of flight coordinates does not involve any physical contact. Due to Newton's third law (the principle of action and reaction, according to which reaction is equal to and opposite to action), any impulse gained by a spacecraft must be lost relative to the planet or vice versa. However, because the planet is much, much more massive than the spacecraft, its effect on the planet's orbit is negligible [9].

The Oberth' effect, in which a rocket engine generates more energy when traveling at high speed than if it is constantly traveling at a lower speed, can be used especially during a gravity assistance

operation. The Oberth effect can be employed, particularly during a gravity assist operation. This effect consists in the fact that the use of a propulsion system works better at high speeds and therefore the course changes are best performed when they are close to a gravitational solid body. This can lead to the multiplication of the effective velocity derivative, i.e. the acceleration.

9. ORBITAL SPECIFIC ENERGY

By conic section is meant a curve resulting from the intersection of a plane with a straight circular cone [8]. As shown in Figure 2, the angular orientation of the plane with respect to the cone determines whether the conical section is a circle, an ellipse, a parabola, or a hyperbola. The circle and ellipse occur when the intersection between the cone and the plane is a bounded or closed curve. The circle is a special case of the ellipse when the plane is perpendicular to the axis of the cone. If the plane is parallel to the cone generator line, then an open curve is obtained called a parabola. Also, if the intersection is an unlimited curve and the plane is parallel to the cone axis or it's not parallel to the cone generator line, then this is a hyperbola. In the latter case, the plane will intersect both halves of the cone, producing two separate curves.

For a given major semi-axis, the specific orbital energy is independent of the magnitude of the eccentricity. In some particular cases:

Table 2.

Conic section	Eccentricity, e	Semi-axis major	Energy
Circle	0	radius	<0
Ellipse	$0 < e < 1$	>0	<0
Parabola	1	infinity	0
Hyperbola	>1	<0	>0

The type of conical sections can be defined in terms of eccentricity. The conical section type is also dependent to the major semi-axis and energy. Table 2 shows the relationship between conical section type in relation to eccentricity, major semi-axis, and energy.

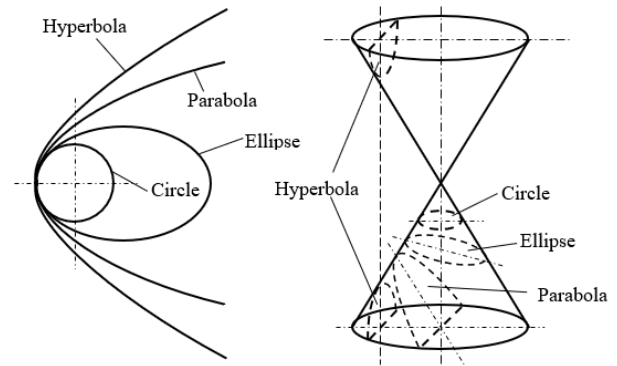


Figure 2.

9.1 Circular trajectory

It is assumed that the position of the solid body is described by a vector \vec{r} relative to the center of Earth C , as shown in Figure 3, which illustrates the initial coordinate system of the solid body [10].

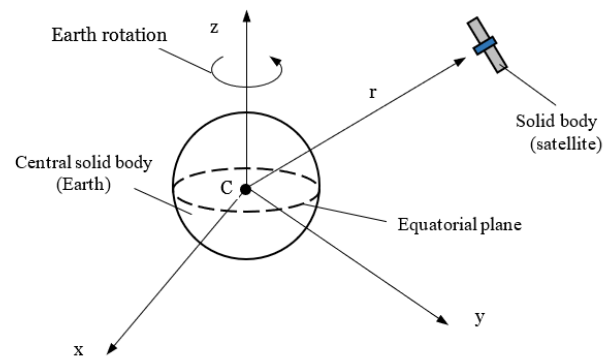


Figure 3.

If the solid body receives an acceleration motion, then a force described by Newton's first law acts on it:

$$F = m \frac{d^2 r}{dt^2} \quad (20)$$

However, the F_c centripetal force given by the relationship (4) also acts on the solid body. Therefore, from the equality of the two forces, it is obtained:

$$-m \frac{\mu r}{r^3} = m \frac{d^2 r}{dt^2} \quad (21)$$

Which can also be written:

$$\frac{d^2 r}{dt^2} + \frac{\mu r}{r^3} = 0 \quad (22)$$

This is a second-order linear differential equation which in practical applications, it is solved for the unknown variable r putting conditions at the limit.

9.2 Elliptical trajectory

The assumption that the satellite follows a circular trajectory is sufficient to calculate the orbital period.

However, in general, solid bodies do not move exclusively on circular orbits around a center of mass C. As shown in Table 2 and Figure 2, the shape of the orbital trajectory of a solid body depends on the eccentricity value e [10].

In the case of an elliptical trajectory, the unknown variable r and its unit vector \vec{r} are time functions. These are in the report:

$$\mathbf{r} = r(t)\vec{r}(t) \quad (23)$$

In order to solve the equation (22) the first and the second derivative, $\frac{dr}{dt}$ and $\frac{d^2r}{dt^2}$, must be determined using the product rule:

$$\frac{dr}{dt} = \frac{dr(t)}{dt}\vec{r}(t) + \frac{d\vec{r}(t)}{dt}r(t) \quad (24)$$

Equation (24) it is resolved by expressing r in a polar coordinate system with a simpler dependence on time and angles. In the polar coordinate system, the orbital plane of the solid body coincides with the x_0y_0 plane. The coordinate system is shown in Figure 4.

Converting Cartesian coordinates into polar coordinates (by cylindrical form) can be done with the following relationships:

$$r_0 = x_0\vec{x}_0 + y_0\vec{y}_0 = r_0\vec{r}_0 \quad (25)$$

$$\vec{r}_0 = \cos\varphi_0\vec{x}_0 + \sin\varphi_0\vec{y}_0 \quad (26)$$

$$\vec{\varphi}_0 = -\sin\varphi_0\vec{x}_0 + \cos\varphi_0\vec{y}_0 \quad (27)$$

With these values, equation (22) becomes:

$$\frac{d^2r_0}{dt^2} + \frac{\mu r_0}{r_0^3} = 0 \quad (28)$$

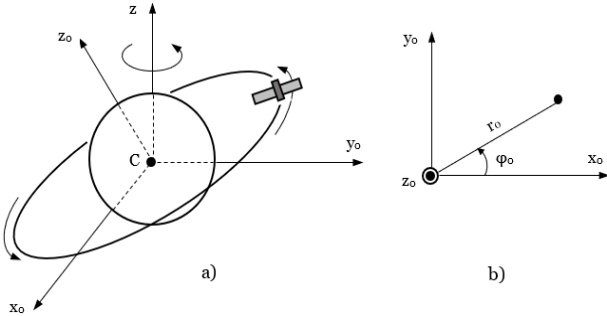


Figure 4.

- a) the polar coordinate system
b) $x_0 - y_0$ plane viewed from above

The solution of this equation has two components: a radial one (\vec{r}_0) and another axial ($\vec{\varphi}_0$). Deriving the left side of the equation (28), the radial component is given by:

$$\frac{d^2r_0}{dt^2} - r_0 \left(\frac{d\varphi_0}{dt}\right)^2 = -\frac{\mu}{r_0^2} \quad (29)$$

and the axial component is:

$$2\frac{dr_0}{dt}\frac{d\varphi_0}{dt} + r_0\frac{d^2\varphi_0}{dt^2} = 0 \quad (30)$$

Dividing the equation (24) with r_0 and applying the product rule for the left-hand side of this equation followed by derivation with respect to the variables r and φ_0 , is obtains:

$$\frac{1}{r_0}\frac{d}{dt}\left(r^2\frac{d\varphi_0}{dt}\right) = \frac{1}{r_0}\left(2r\frac{d\varphi_0}{dt} + r_0^2\frac{d^2\varphi_0}{dt^2}\right) \quad (31)$$

Is observed that the right hand side of equation (31) is similar to equation (30). It follows from this that:

$$\frac{1}{r_0}\frac{d}{dt}\left(r^2\frac{d\varphi_0}{dt}\right) = 0 \quad (32)$$

which means that:

$$r^2\frac{d\varphi_0}{dt} = \text{constant} = h \quad (33)$$

where h is the angular momentum per the mass unit [m^2/s] or [N m s].

On the other hand, solution of the equation (29) can be of the form:

$$r_0 = \frac{h^2}{\mu + Ah^2\cos(\varphi_0 + \theta_0)} \quad (34)$$

where A is an integration constant and θ_0 [rad] is the angle covered by the solid body during t [sec]. This equation can be written as:

$$r_0 = \frac{\frac{h^2}{\mu}}{1 + \frac{Ah^2}{\mu}\cos(\varphi_0 + \theta_0)} = \frac{p}{1 + e\cos(\varphi_0 + \theta_0)} \quad (35)$$

which represents the equation of an ellipse in polar coordinates. The values of $p = \frac{h^2}{\mu}$ also represent the semilatus rectum (right half) of the ellipse [m], and $e = \frac{Ah^2}{\mu}$ is the eccentricity of the ellipse [m]. It can eliminate θ_0 by aligning the x_0 axis of the polar coordinate system and its coincidence with the major semi-axis of the ellipse, and is obtain:

$$r_0 = \frac{p}{1 + e\cos\varphi_0} \quad (36)$$

The orbital path in the polar coordinate system is illustrated in Figure 5. The solid body moves on an elliptical trajectory relative to the origin of the coordinate system C . The ellipse's outbreaks are located at points F_1 and F_2 . In the example of Figure 5, the Earth is located at the focal point F_2 . This is the first of three laws of Kepler's planetary movement: the orbit of a smaller solid body relative to a larger solid body is always an ellipse, with the center of mass of the larger solid body coinciding with one of the two focal points of the ellipse.

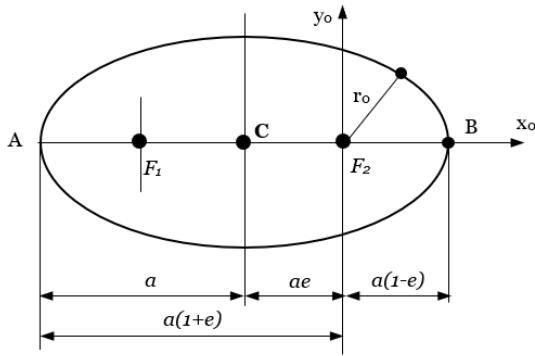


Figure 5.

The length of the major semi-axis of the ellipse is:

$$a = \frac{p}{1-e^2} \quad (37)$$

while the length of the minor semi-axis is:

$$b = a\sqrt{1-e^2} \quad (38)$$

It is assumed that a solid body, such as the Earth, is located at the focal point F_2 . Another solid body of smaller size, such as a satellite, crosses the orbital path and reaches in points A and B. These points are the farthest, or closest to Earth, respectively.

10. ORBITAL SPECIFIC ENERGY

To describe mathematically an orbit, six parameters must be defined, named orbital elements. These are [2, 3 and 3]:

- Without applying an external force, such as the firing force emitted by a rocket engine, the period and shape of the satellite's orbit will not change.
- A satellite that gravities on a low orbit or on the low part of an elliptical orbit, moves faster than the planet's reference surface relative to another satellite that revolves over a larger orbit or on the higher part of an elliptical orbit, due to higher gravitational field of the satellite located closer to the planet.
- If the external force is applied only in one point from the satellite orbit, the satellite will return to the same point on each subsequent orbit, although its trajectories will always change. Consequently, one cannot move from one circular orbit to another with a single brief application of a thrust pulse.
 - Major semi-axis, a
 - Eccentricity, e
 - Inclination, i
 - Argument of periapsis, ω
 - Time of periapsis passage, T
 - Longitude of ascending node, Ω

An orbital satellite follows an elliptical trajectory, and the center of the planet, called the central solid body, is located at one of the two focal points. An ellipse is defined to be a closed curve having the

following characteristic: for each point on the ellipse, the sum of its distances to the focus points is constant (see Figure 6).

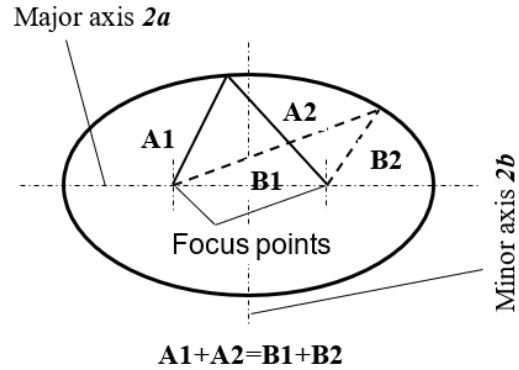


Figure 6.

The longest and shortest lines that can be drawn through the center of an ellipse are called the *major* (the main) *axis* and *minor* (secondary) *axis*, respectively.

The *major semi-axis* a , is half of the main axis and represents the mean distance of the satellite relative to one of the focus points.

Eccentricity e is the distance between the focal points divided by the length of the main axis and has a value between zero and one. An eccentricity equal to zero indicates a circle.

Inclination i , is defined to be the angular distance between the satellite's orbital plane and the equator of its first plane (or elliptical plane in the case of heliocentric or sun-centered orbits). A zero-degree inclination indicates an orbit around the equator of the central solid body in the same direction as its rotation, a direction called *prograde* or directly. A 90 degree inclination indicates a *polar orbit*. An inclination of 180 degrees indicates a *retrograde equatorial orbit*. A retrograde orbit is one in which a satellite moves in a direction opposite to the rotation of its planet.

Periapsis is the point on an orbit closer to that of the central solid body. The opposite of periapsis, that is the most distant point on an orbit, is called *apoapsis*. Periapsis and apoapsis are usually modified to apply to the orbital body reported to the central planet, such as *perihelion* and *aphelion* for the Sun, *perigee* and *apogee* for Earth, *perijove* and *apojove* for Jupiter, *perilune* and *apolune* for the Moon etc.

The *periapsis argument* ω is the angular distance between the ascending node and the periapsis point (see Figure 7).

The *time of periapsis passage* T is the moment when a satellite moves through its point of periapsis.

Nodes represent the points in which an orbit crosses a plane, such as a satellite or another solid body that crosses the equatorial plane of the Earth. If the satellite or a solid body crosses the plane that goes from south to north, the node is called as *node ascending* (N_1); if it moves from north to south, it is a *descending node* (N_2).

Longitude of ascending node Ω is the celestial longitude of the node. The celestial longitude is analogous to the Earth's longitude and is measured in degrees counter-clockwise from zero, where the zero point is in the direction of the vernal equinox.

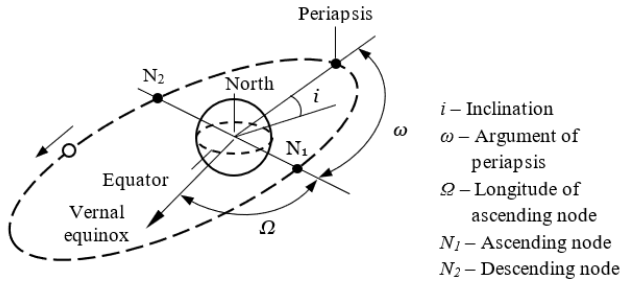


Figure 7.

Generally, three observations of a solid body in orbit are needed to calculate the six orbital elements above. Two other parameters are often used to describe the orbits and are represented by true anomalies of the period. *Period P*, is the length of time required for a satellite to go through an orbit. *True anomaly* is the angular distance of a point (center of mass of a solid body) on an orbit that has passed through periapsis and is measured in degrees.

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