THE PROCESSING OF HYPERBOLOID SURFACES USING WIRE ELECTRICAL DISCHARGE MACHINING – THEORETICAL ASPECTS

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ABSTRACT: The paper presents an original method that can be used for cutting hyperboloid surfaces using wire electrical discharge machining (WEDM). The processing method, which has only been partly revealed by the author in certain relatively low-impact Romanian publications, is now fully delineated in the form of two papers that focus on both the theoretical and practical aspects of the processing of hyperboloid surfaces using wire electrical discharge machining. This paper presents the theoretical facets of the matter, revealing the mathematical aspects that underpin the methods of generating hyperboloid surfaces. The way in which a hyperboloid surface can be materialized for any mathematical equation of a hyperbola, which defines the axial profile of the hyperboloid of revolution that can be generated using wire electrical discharge machining is minutely explained, thus ensuring the availability of all information required by the practical implementation of the processing.
KEY WORDS: WEDM, hyperboloid surfaces, theoretical aspects

1. GENERALITIES

Given that surfaces that take the shape of hyperboloids of rotation can be obtained by using a slanted generating line swept about a circular trajectory (A hyperboloid surfaceFigure 1), the possibilities of generating this type of surfaces have been delved into.

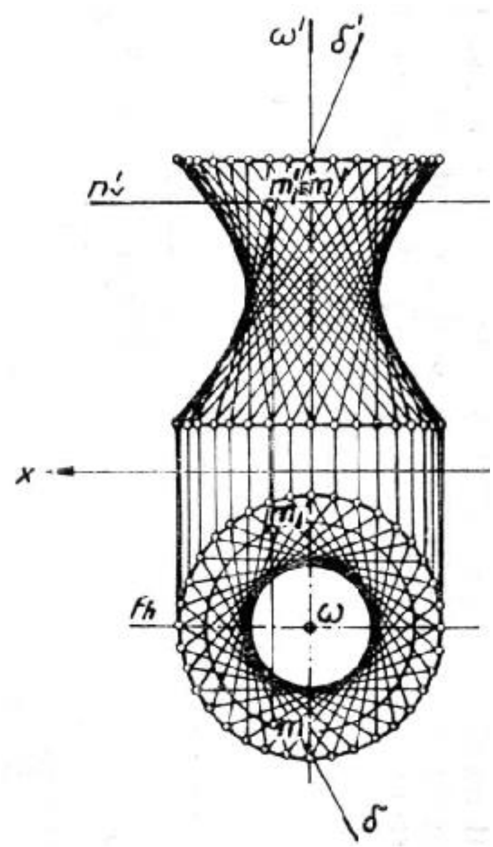


Figure 1. A hyperboloid surface

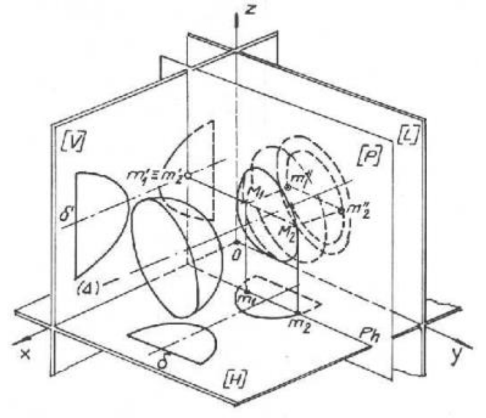
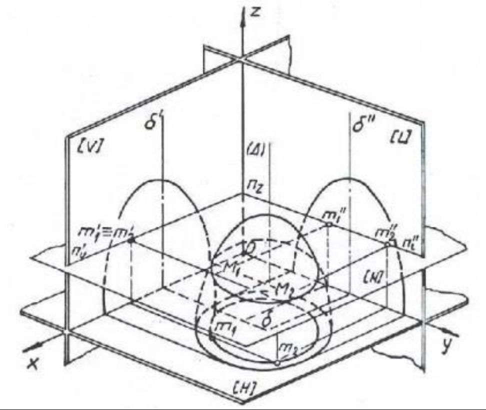
The processing of shafts and bores that include hyperboloid surfaces also raise certain technological issues, concerning the ways of removing the resulting solid debris (some solutions are presented in [1]).

Globally there has only been one mention so far concerning the processing of such surfaces [2], but there are no accompanying theoretical or practical considerations put forth with reference to the way in which the processing can be achieved. It is for this reason that some research into the mechanism of generating this type of surfaces is very welcome, together with the theoretical outlining of the process and its practical implementation.

2. DETERMINING THE LAWS OF MOTION WHEN GENERATING HYPERBOLOID SURFACES

There are infrequent cases of parts that contain hyperboloid surfaces in practice. There are two categories of hyperboloid surfaces used to describe hyperboloid bodies: the two-sheet hyperboloid of revolution and the one-sheet hyperboloid of revolution, both of which are shown in Table 1.

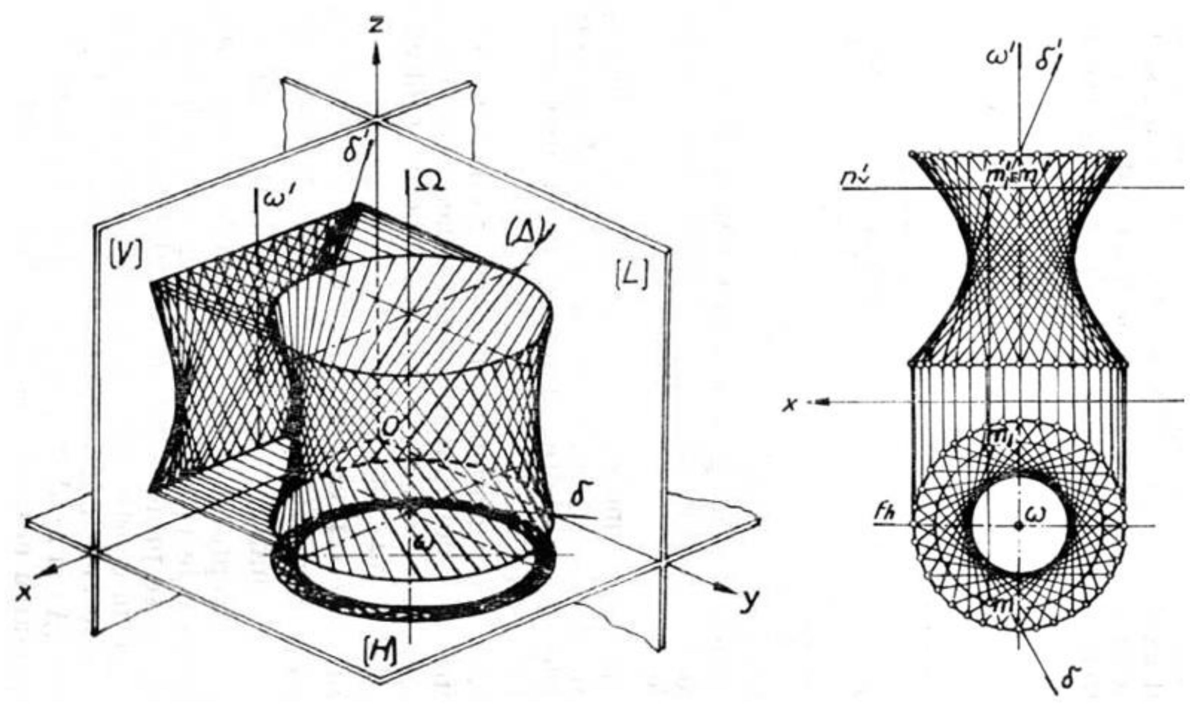
Table 1. Types of hyperboloid surfaces

Type of surface	Shape of surface	Method of generation
Two-sheet hyperboloid of revolution		A) Revolving a hyperbola about its transversal axis
One-sheet hyperboloid of revolution		A) Revolving a slanted line about an axis which is non-coplanar with the line or: B) Revolving a hyperbola about a non-transversal axis (an axis which is coplanar with the hyperbola, orthogonal to its transversal axis, and that goes through its centre)

The infrequent usage of hyperboloid surfaces is in part due to the technological difficulties concerning the generation of this type of surfaces. Taking however into account the specificity of WEDM processing, and of the method of generating hyperboloid surfaces, which is shown in Figure 1, it can be claimed that the generation of one-sheet hyperboloid surfaces is possible, which – at least theoretically – holds true. In spite of all this, no specialty paper or manufacturer's brochure has ever presented the possibility of generating hyperboloid surfaces using WEDM.

While it is true that the generation of any type of surface is developed only once its usefulness has been proven, the issue can be seen from another perspective as well. Once defined from a theoretical and technical perspective, the process of generating a type of complex surface can become appealing to potential beneficiaries, especially if the processing costs are low. This is why the theoretical and practical bases of generating hyperboloid surfaces using WEDM are going to be delineated over the next pages, especially due to the fact that this type of processing (i.e. WEDM) is experiencing a dynamic evolution and becoming more and more efficient at the same time – both from a technical, and an economic standpoint.

It has been shown – in Table 1 – that there are two ways of generating one-sheet hyperboloid surfaces. The alternative in which this type of surface is achieved using a generating line revolving about an axis which is non-coplanar with the line yields a surface which is similar to the one shown in Figure 2, where each point on the line generates circles which are orthogonal to the revolution axis.



a. b.

Figure 2. A hyperboloid of revolution

Of these circles, the smallest diameter circle is called a *collar*. The radius of the collar circle is the length of the common normal to the generating line and the revolution axis. When projected onto a plane which is orthogonal to the revolution axis, the generating line remains always tangent to the collar circle.

The definition of a hyperboloid surface is achieved more easily if we employ the second generation method, by revolving a hyperbola about a non-transversal axis (case B) from Table 1).

To do this, let us consider a hyperbola [H] which, in relation to reference place $y_1O_1z_1$, is characterized by the following general equation:

$$\frac{y_1^2}{a^2}-\frac{z_1^2}{b^2}-1=0 \qquad (1)$$

Where *a* and *b* are the lengths of the semiaxes and $c=\sqrt{a^2+b^2}$ is the distance from its centre, O_1, to the focus, F, as shown in Figure 3. It can be noted that semiaxis a is none other than the radius of the collar circle (r = a). For the purposes of the following analysis, only a portion of the generated hyperboloid of revolution will be considered, i.e. the one bounded by two planes normal to axis O_1z_1 and equally spaced (at distance h) to the centre of the generating hyperbola (O_1). Consequently, the delimited surface is symmetrical, the ends of which are bounded by R - radius circles. This R radius represents the greatest value of a circle described by a point on the generating hyperbola on the delimited region.

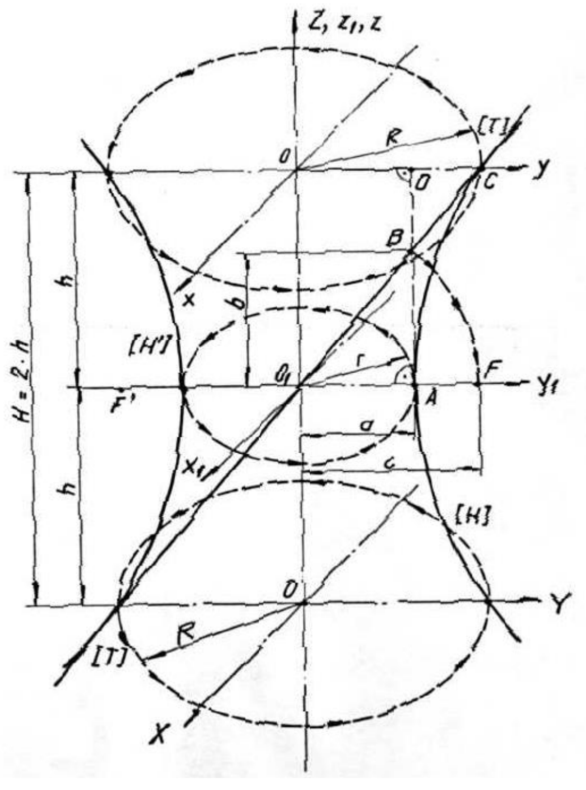


Figure 3. Defining a hyperboloid surface

If the semiaxes of the hyperbola are equal, i.e. a = b, the specific hyperbola obtained is equilateral, and its equation – with reference to the same plane, $y_1O_1z_1$, becomes:

$$y_1^2 - z_1^2 = a^2 \text{ , or: } y_1^2 - z_1^2 = r^2 \qquad (2)$$

Of some interest is the general equation of the tangent to the hyperbola [H] in a point on the hyperbola, whose coordinates are (y_0, z_0):

$$\frac{y \cdot y_0}{a^2} - \frac{z \cdot z_0}{b^2} - 1 = 0 \qquad (3)$$

The same hyperboloid surface can also be generated by revolving a slanted line about an axis which is non-coplanar with the line, as described in Table 1 and illustrated in Figure 4.

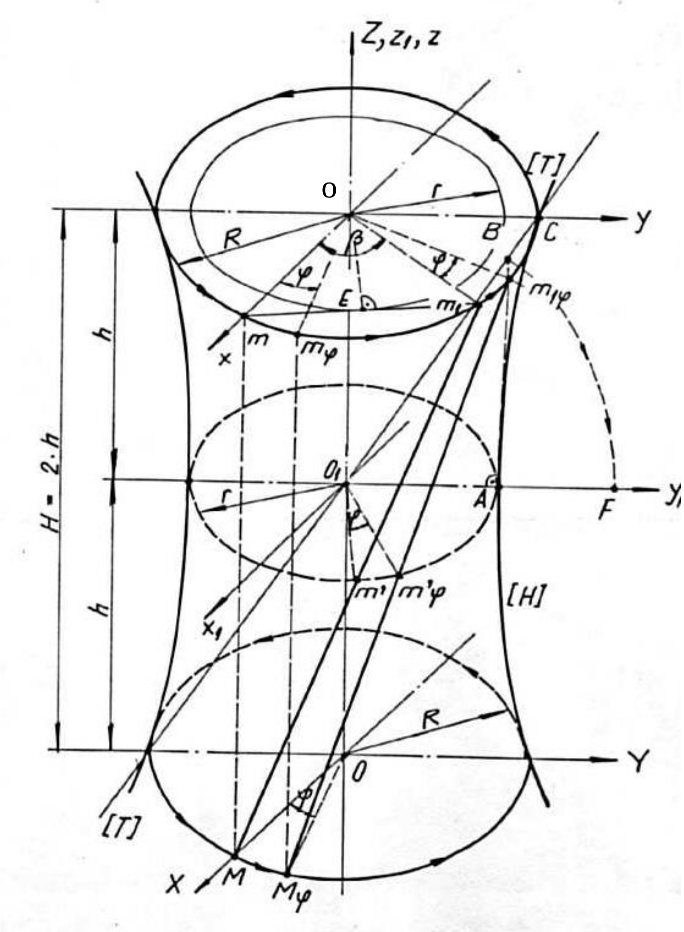


Figure 4. Generating a hyperboloid surface

If the generating line were to occupy the initial position defined by the line segment Mm, and if it were to revolve around a vertical axis (OZ), with which it is coplanar, then it would generate a cylindrical surface whose radius is R. In order to generate a hyperboloid surface, it is necessary that the generating line be slanted to the vertical axis. If the line in question were to be defined by the lower point $M \in XOY$ and by the upper point $m \in x_1O_1y_1$, then it becomes slanted and no longer coplanar with the vertical axis. In this case, by rotating line Mm_1 about axis OZ, a hyperboloid revolution surface will be generated. It can be noted that line Mm_1 generates R - radius end circles. At the same time, the initial slanting of the generating line is achieved by moving point $m \in x_1O_1y_1$ to a new position, m_1, defined by the R - radius upper circle and the central angle β, which will be hitherto referred to as displacement angle.

Throughout the generating process, the generating line will occupy consecutive $M'm_1'$ positions, defined by central angle α. If α is swept across the entire [0, 2π] interval, the generating process of the hyperboloid of revolution is complete.

In order to generate the same hyperboloid surface through each of the two methods of generation laid out in Table 1, we must calculate the displacement angle β that ensures the fulfilment of this condition. It is essential to note the projection of line Mm_1 onto plane xoy, as shown in Figure 4. This projection is none other than segment mm_1. Since right-angled triangle oEm (which is similar to triangle oEm_1) is such that oE = r and om = R, it results that the displacement angle β is defined by:

$$\beta = 2 \text{ arc cos } \frac{r}{R} \qquad (4)$$

Since r = a, and using equation (1), it results that it is enough to specify R based on the semiaxes of the generating hyperbola [H]. It becomes self-evident that the relation sought will also depend on distance h, which also defines the vertical position of the extremities of the hyperboloid surface considered.

To solve the problem, we must use the equation of the tangent belonging to plane YOZ to the hyperbola [H] at point C (R, h) by solving for the parameters in equation (3):

$$\frac{y_1 \cdot R}{a^2} - \frac{z_1 \cdot h}{b^2} - 1 = 0 \qquad (5)$$

which, when we assign the values for y_1 and z_1, for the same point considered, it results that:

$$R = \frac{\sqrt{a^2 \cdot h^2 + a^2 \cdot b^2}}{b} = \frac{a}{b} \cdot \sqrt{h^2 + b^2} \qquad (6)$$

The same result is reached if we input the coordinates of the same point (i.e. C (R, h)) into general equation (1).

As a result of this, the displacement angle β will be:

$$\beta = 2 \cdot arc\ cos \frac{b}{\sqrt{h^2 + b^2}} \qquad (7)$$

If we are dealing with the generation of a hyperboloid surface whose axial section is an equilateral hyperbola, the displacement angle becomes:

$$\beta = 2 \cdot arc\ cos \frac{a}{\sqrt{h^2 + a^2}} \qquad (8)$$

We have thus managed to define the initial position of generating line Mm_1 which in turn can define –

by revolving it around revolution axis OZ – a hyperboloid surface which is of interest only for a random finite height equal to 2h.

A common trait of both generation methods is that the same generating curve [Δ] – which is represented by a circle – is used. Concerning the generating curve [Γ], things are different: in one instance it is a hyperbola, and in another a line. Table 2 shows the specific elements of the generation methods of limited hyperboloid surfaces.

Table 2. Ways of generating limited hyperboloid surfaces

<table>
<tr><th>Generation methods</th><th>Specific elements</th><th>Controllable geometric elements</th></tr>
<tr><td>A) Revolving a slanted line about an axis which is non-coplanar with the line</td><td>- generating curve [Δ]: R-radius circle
- generating curve [Γ]: slanted line defined by displacement angle β
$\beta = 2 \cdot arc\ cos \frac{b}{\sqrt{h^2 + b^2}}$</td><td rowspan="2">- specific diameters: d = 2r; D = 2R
- height: H = 2h
- profile of transversal section: circle
- profile of axial section: arc of hyperbola
- other quality-related characteristics: based on specific requirements</td></tr>
<tr><td>B) Revolving a hyperbola about a non-transversal axis</td><td>- generating curve [Δ]: R-radius circle
- generating curve [Γ]: hyperbola of this equation:
$\frac{y_1^2}{a^2} - \frac{z_1^2}{b^2} - 1 = 0$</td></tr>
</table>

It is worth mentioning that, in the case of WEDM, we can only apply the generation method that requires the use of a generating line, given that this curve must be followed by the wire electrode used for processing.

The specific equations of the hyperbola that define the axial section of a hyperboloid of revolution are of great use. Based on the notations made in Figure 3, by applying equations (1) and (3), solved for point C (R, h) situated on plane $y_1o_1z_1$, the equation of hyperbola [H] can be written as:

$$z_1 = \pm h \sqrt{\frac{y_1^2 - r^2}{R^2 - r^2}} \qquad (9)$$

if we solve for variable y_1, or:

$$y_1 = \frac{1}{h} \sqrt{z_1^2 (R^2 - r^2) + h^2 \cdot r^2} \qquad (10)$$

if we solve for variable z_1.

Both previous equations can be used to define the hyperboloid, and also to control such surfaces.

3. CALCULATING THE LAWS OF MOTION WHEN GENERATING HYPERBOLOID SURFACES

Since the generating movements required when processing any type of surface by using WEDM are relative, these can be made either by: the workpiece being processed, the wire electrode, or both of these. The last setup is the most difficult to implement. Conversely, the first two setups are readily applicable in practice.

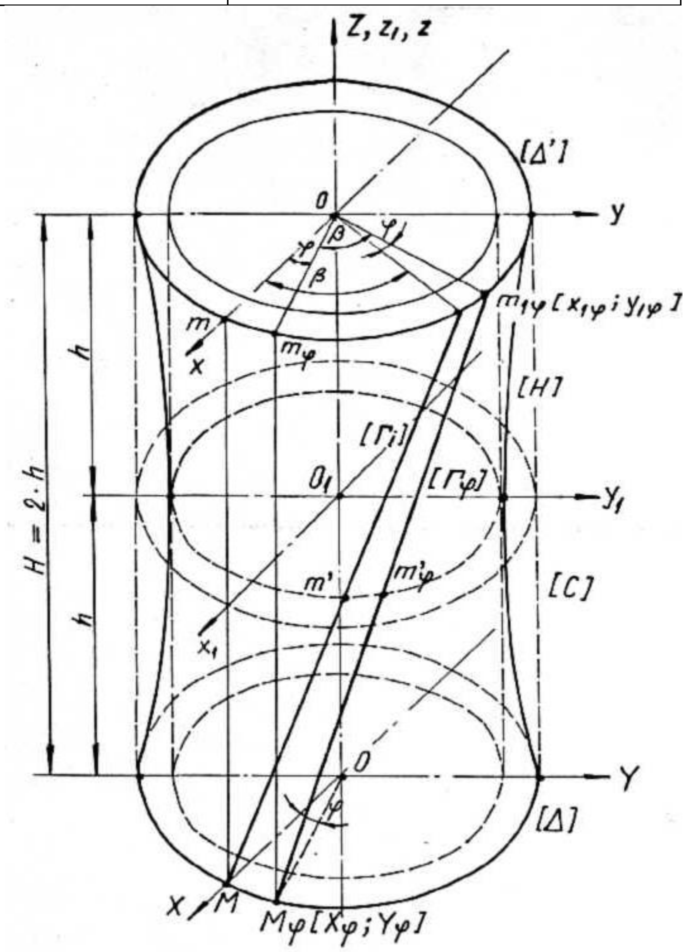


Figure 5. Defining the position of the current generating line

If the generation movements are made by the workpiece, then it becomes necessary to use the patented generating device presented in [1], which allows the workpiece to make two rotations about two mutually perpendicular axes. When the generation movements are made by the wire electrode, we can use modern equipment which allows the command and control of at least four coordinate axes.

Since most new machines are capable of making slanted cuts, having control over 4 or more axes, we must wonder whether it is possible to generate hyperboloid surfaces with their aid, especially given that neither the specialty literature nor the manufacturers' brochures hint at the existence of such a technological possibility. Consequently, the paper broaches an issue that hasn't been studied hitherto, either nationally or internationally, there being only one article in the field [2], as stated at the beginning of this paper.

The slanting possibilities of the wire electrode of modern equipment allow the implementation of the hyperboloid surfaces' generation by revolving a slanted line about an axis which is non-coplanar with the line, as described in Table 1. The only problems that arise concern the impossibility of exceeding the maximum slanting angle of the wire electrode (which is typically ±30°) and the fact that the CNC capabilities of these machines are not especially tailored to also control generation movements of the type that are required when processing hyperboloid surfaces.

In order to process hyperboloid surfaces, we must first slant the wire electrode so that this occupies the initial position of the generating curve $[\Gamma_i]$, defined by the line segment Mm_1 (Figure 5). The ends of the line segment have coordinates that can be determined on planes delimiting the height of the hyperboloid surface generated.

Thus, point $M \in XOY$ is situated on the lower generating curve $[\Delta]$, and is defined by the coordinates M [R, 0]. The other extremity, $m \in xoy$ is situated on the lower generating curve $[\Delta']$, and is defined by the coordinates $m[R \cdot \cos \beta, R \cdot \sin \beta]$. For the purposes of the processing, it is sufficient that the two extremities of the initial generating curve $[\Gamma_i]$ follow the two generating curves, $[\Delta]$ and $[\Delta']$, at the same speed. If the movements span the entire length of the generating curves, the hyperboloid surface will be fully generated.

The matter can be described mathematically by the position of the current generating curve $[\Gamma\varphi]$, defined by the random instantaneous angle φ, as can be seen in Figure 5. Its extremities need to follow the two circles ($[\Delta]$ and $[\Delta']$) of identical radii (R). The movements of the two extremities are phase-shifted at precisely the displacement angle β, whose definition is given by equation (7). Thus, the laws of motion of the current lower point $M\varphi[X\varphi, Y\varphi] \in [\Delta]$ are:

$$X\varphi = R \cdot \cos \varphi \text{ , } \varphi \in [0,2\pi] \qquad (11)$$

$$Y\varphi = R \cdot \sin \varphi \text{ , } \varphi \in [0,2\pi] \qquad (12)$$

whereas the current upper point $m_1\varphi[x_1\varphi, y_1\varphi] \in [\Delta']$ will be characterized by these expressions:

$$x_1\varphi = R \cdot \cos (\varphi+\beta) \text{ , } \varphi \in [0,2\pi] \qquad (13)$$

$$y_1\varphi = R \cdot \sin (\varphi+\beta) \text{ , } \varphi \in [0,2\pi] \qquad (14)$$

Even though modern machines allow the generation of revolution surfaces characterized by different upper $[\Delta']$ and lower $[\Delta]$ generating curves, these do not afford the possibility of following the same type of generating curves that have different or phased starting points, as is required in order to facilitate the generation of hyperboloid surfaces. It is therefore sufficient to append this additional functionality – in the form of software applications that enable the generation and post-processing of the required motions – to the CNC equipment. This constitutes a solvable automation issue that WEDM machinery manufacturers must solve, so as to offer new technological generation possibilities.

4. CONCLUSIONS

The processing of hyperboloid surfaces using WEDM is possible with the aid of the mathematical apparatus herein exhaustively outlined as a world premiere. Modern WEDM machines can be programmed to process hyperboloid surfaces by making some simple and doable updates. The processing of hyperboloid surfaces using WEDM is likely to become a future facility that WEDM machines can offer.

5. REFERENCES

1. Deneș, C. *Contribuţii asupra prelucrării prin eroziune electrică cu electrod filiform*. Diss. Universitatea "Lucian Blaga" din Sibiu, Romania, (2002).
2. *** *EDM Today*. EDM Publications, Nov.-Dec. 2000.