

# OVERVIEW OF ANALYTICAL MODELING IN LASER CUTTING PROCESSES

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**ABSTRACT:** Main elements of the analytical modelling are presented for the laser cutting process. Analytical models consider the physical phenomena that occur and formulate balance equations. In current approaches, analytical modelling is integrated into simulation using software dedicated to this purpose. This moves away from the formation and knowledge of elementary models. The technological applicability of analytical modelling is reduced compared to the empirical one, through correlation formulas, because the analytical model is either too general or too particular. The sequential treatment of a complex processing process and the modelling of physical phenomena that have a significant contribution can be a way to exploit analytical modelling.

**KEYWORDS:** laser cutting, evaporation, fusion, analytical modelling

## 1. INTRODUCTION

Mathematical modelling plays an important role in understanding complex processing processes. *Analytical modeling* commences by recognizing the underlying physical phenomena and their translation into mathematical expressions using balance equations (such as energy or mass equations). On the other hand, empirical modeling takes a distinct route, establishing connections between objective function values and the influencing factors through mathematical relationships. The physical phenomena that occur are ignored, but a statistical study of the adequacy of the proposed models is carried out.

Empirical modelling quickly responds to some technological objectives such as, as an illustration, consider factors such as the extent of the thermally affected zone and the width of the resulting cut (representing the volume of material removed), the analysis of the quality of the machined workpiece. Analytical modelling gives an indirect answer to these problems because it remains in general assumptions that most often do not coincide with experimental conditions. Also, not every possible physical phenomenon takes place, and if it does, it does not mean that it has a significant contribution. However, there is a method to improve the applicability of analytical modelling. It involves a process of sequencing the physical phenomena that take place and the separate modelling of some physical phenomena through simple balance equations Boboescu and Herman [2].

The paper presents the phenomena involved in laser cutting, approached from the point of view of analytical modeling. The aim was to present the physical phenomena involved, how they are considered in the

model and the resulting equations, as well as analyzing the results obtained by different authors.

The graphs obtained based on the analytical modeling show the following categories:

- correlations between process parameters to establish optimal combinations.
- correlations between parameters and objective functions, as an illustration, the width of the cut is contingent upon the laser beam's power.
- the representation of a physical space, including instances like the cutting edge and surface cut piece striations.

Analytical modeling starts from formulating some assumptions of the model. They allow the formulation of differential equations. Their solution in most cases is numerical.

There are two distinct approaches here:

- Simple (one-dimensional) models that allow the analytical solution of differential equations.
- Complex (three-dimensional) models that require numerical solution.

In practice, both ways must be approached, and the numerical solution must be appropriated by an analytical one based on the simplified model.

Analytical models must have a certain generality. For this, the objective functions must describe the process globally. Hence, an analytical model can effectively depict the volume of material removal but lacks the capability to predict the width of the cut. Consequently, these models tend to be more intricate to apply compared to empirical ones.

The erosion front is precisely characterized as isothermal, maintaining a constant temperature at the melting point. The model also delves into the

dynamics of its movement and the expulsion of molten material in the form of droplets. An extensively discussed issue in modeling pertains to the thickness and temperature of the molten material layer. This document presents several facets associated with this matter, leading to the formulation of specific conclusions. In connection with "Cutting speed", the study investigates the maximum achievable cutting speed for a given laser power and a specified material thickness.

The contributions of evaporation and melting to the increase in the cutting speed are also addressed, as well as the problem of the appearance of sparks at the base of the kerf. The angle at which they scatter is connected to the cutting speed.

An essential model pertains to the kinetics of melting, specifically, the movement of the solid-liquid interface, which is addressed in a one-dimensional approach by Kapat and Kumar [8]. The research refers to presenting the differences that are obtained in terms of the time variation of the temperature at the interface and its position if we use the Stefan condition at interface (based on the equality of temperatures) or the kinetic condition (based on the free energy variation in the phase transformation).

The thermally affected zone means the zone where a significant heating of the material takes place, affecting its properties. It is described by the conduction of heat in the material starting from the secondary heat source represented by the cutting front. For this, the heat equation is applied.

The dynamic attributes of the gas jet in laser cutting represent a significant modeling challenge, particularly concerning the gas jet's propagation. This involves an examination of factors related to the gas jet's configuration, its spread, and its efficacy in eliminating molten material. Additionally, scenarios involving both subsonic and supersonic flows are studied, with a focus on simulating the gas jet's shape and flow patterns. The consequences of pressure gradients within the gas jet are potential issues such as burring, constraints on cutting depth, and laser beam defocusing.

The appearance of striations on the surface of the cut can also be explained based on gas-dynamic phenomena. For fusion laser cutting the role of the gas jet in cutting and its action at the erosion front, the influence of the gas jet on the cutting speed and the thickness of the liquid layer at the interface are presented.

The gas jet has the role of removing the melted and vaporized material and protecting the focusing lens, and if it is a reactive gas, it brings energy to the erosion front. The main problem to be addressed by the model is the role of the gas jet speed variation. It is shown that no matter how much the speed of the gas jet increases at the erosion front, the molten material cannot be completely removed. Fusion laser cutting mainly refers to the case where the burning reaction of iron takes place Yilbas and Sahin [9].

The gas used is oxygen. This reaction is thermally initiated, the heat absorbed from the laser having this role. The reaction is exothermic and brings energy to the erosion front. The open problem is the appearance of striations on the walls of the cut. These are either attributed to the oscillations of the erosion front, or they are explained as flow lines of the molten material. Laser cutting of ceramic materials occurs by fracturing the material. Another problem addressed is the establishment of an optimal cutting speed and an empirical way by which it can be related to the thickness of the sample.

The paper aimed to present those aspects that can form an image of laser cutting, the physical phenomena that appear in it and the main problems that are the object of analytical modeling in laser cutting. Modeling laser cutting processes introduces novel technological prospects for enhancing efficiency and achieving substantial economic benefits.

## **2. CLASSIFICATION OF LASER CUTTING PROCESSES**

Laser cutting is classified according to the way the material is removed:

- a)** with the complete removal of the material along the separation surface:
  - by vaporization;
  - by melting and limited vaporization;
  - with the autonomous expulsion of the material from the cutting area.
  - with the expulsion of the material from the cutting area through the erosive action of a gas jet blown coaxially with the laser beam.
- b)** with the partial removal of the material along the separation surface:
  - by vaporizing the material to a depth adequate to achieve an imposed notch effect and the separation following laser irradiation under the action of a mechanical stress, cutting uses a thermo-mechanical mechanism in this case.
  - by vaporizing the material layer deposited on a material support of a different nature

c) without removal of material along the separation surfaces:

- by controlled breaking in the solid state of the material under the action and during the laser irradiation, the laser beam induces a mechanical effect by local dilation and contraction of the material.

- by controlled breaking in the solid state of the material under the action of a mechanical stress applied after laser irradiation – this initiates or favors mechanical breaking.

The first step in analytical modeling of laser cutting is to classify the machining process into one of the types shown above. Obviously, the interest is in the processing processes in which the laser irradiation has the main contribution to the removal of the material.

### 3. EVAPORATION LASER CUTTING

The physical phenomena involved in evaporation laser cutting are the absorption of electromagnetic radiation by the solid material; heating the solid material until it reaches the melting temperature, respectively the vaporization temperature; removal of solid material by sublimation or in the form of liquid droplets. As collateral phenomena appear the conduction of heat in the solid and in the surrounding environment as well as the interaction between the resulting vapors and the incident laser radiation.

As is known, a gas jet is used to remove vapors and the liquid layer formed at the erosion front.

The purpose of modeling is either to explain the phenomena that take place, or to evaluate some sizes of phenomenological or technological interest.

Through modeling, the following aspects are followed: temperature distribution in the solid and the shape of the kerf or groove influenced by the material constants and the cutting speed (scanning speed) the geometric form of the erosion front, its propagation, the thickness of the layer of melted material formed at the erosion front; the evolution of the kerf in time. The models start from the establishment of some initial hypotheses within which the considered phenomena are selected.

The phenomena that produce a significant effect are considered, the absorption of the laser radiation is considered on the surface and the result of the interaction between the laser radiation and the material exclusively the thermal effect. Modeling assumptions also establish parameters by which a phenomenon is considered.

We notice a separation between the parameters that characterize the laser beam, its absorption, reflection, transmission in the material and the parameters involved in the thermal process.

Thus, in the end, the laser leads to the appearance of an internal source of heat in the material that will cause thermal processes.

After the assumptions and parameters are established, the equations that describe the processes are obtained. These are differential equations, such as the heat conduction equation. The next stage is the transformation of the equations into dimensionless equations, they contain variables related to the coefficients. Their form is mathematically approachable. Their solution is carried out using numerical or analytical methods. We will note the following stages after Abakians and Modest [1]:

- establishing a geometry of the irradiated area
- formulation of the energy balance;
- solving the resulting differential equations.

The cutting front through its normal is established by the relation:

$$\vec{n} \cdot (k\nabla T) = 0 \quad (1)$$

$k$  - thermal conductivity;  $T$  - temperature.

The energy balance is written for each of these areas Kim [3], [4].

$$q_L + q_k = q_h + q_{ig} \quad (2)$$

The heat received from the laser  $q_L$  plus the heat due to conduction  $q_k$  equals the heat lost by convection  $q_h$  plus the heat lost by evaporation  $q_{ig}$ .

The energy balance can be considered:

- instantly without variations in time (the previous case)

- with the time variation of the phenomena - we thus have the heat equation

$$\rho \cdot c \left( \frac{\partial T}{\partial \tau} + U \frac{\partial T}{\partial x} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \quad (3)$$

$\tau$  - time [s];  $\rho$  - density [kg/m<sup>3</sup>];  $c$  - specific heat [J/kg·K];  $U$  - cutting speed.

The phenomenon of taking up heat in the material by conduction and bringing it to the vaporization or melting temperature at which erosion occurs was studied in the work Prusa, Girish, and Molian [7]. The conductive mechanism cannot explain the exceeding of the vaporization or melting temperature at the workpiece surface.

Several other assumptions are applied. These would be:

- the processed material is semi-transparent; this means that the laser beam is absorbed in the depth of the material. An extinction coefficient of the laser beam appears. This assumption applies to wax or polymeric materials.

- laser radiation propagates and absorbs through a porous medium. This is an assumption specific to evaporation laser cutting.

The evaporating material constitutes a porous medium, i.e., a medium with variable density.

Thermal coefficients appear in the models and the values of which are not always well established for the particularities of the material and the temperature at which it is being worked on.

Modeling results are presented using relative sizes that can be related to process parameters laser power, cutting speed. The modeling results depend on the numerical methods used to solve the differential equations. The results are presented in the form of diagrams between non-dimensional sizes that correspond to those considered when establishing the equations. They are usually accompanied by diagrams showing the convergence study of the applied numerical method.

The following presents the results obtained Abakians and Modest [1] The parameters involved in the obtained results are:

$N_e$  – characterizes the action of the laser.

$N_k$  – characterizes the conduction in the material.

$U$  – characterizes the cutting speed related to energy diffusion

$$\tau = \beta_s \cdot R \quad (4)$$

$\tau$  - dimensionless extinction coefficient;  $\beta_s$  - extinction coefficient;  $R$  - the radius of the laser beam at piece surface [m]

$$\xi = \frac{x}{R}, \eta = \frac{y}{R}, \zeta = \frac{z}{R} \quad (5)$$

$\xi, \eta, \zeta$  - the dimensionless x, y, z coordinates.

They have the expressions:

$$N_e = \rho_s \cdot h_r \cdot \frac{u}{F_0} \quad (6)$$

$F_0$  - the flux in the center of the laser beam [W/m<sup>2</sup>];

$h_r$  - the heat required to remove the mass unit [J/kg];

$u$  - cutting speed [m/s];

$k$  - thermal conductivity [W/m·K].

$$N_k = \frac{k(T_{ev} - T_{\infty})}{R \cdot F_0} \quad (7)$$

$$U = \rho_s \cdot \tau \cdot \frac{uR}{k} \quad (8)$$

$\rho_s$  - the density of the solid [kg/m<sup>3</sup>].

The vaporization cutting model has the advantage of being simple in the meaning that there is only one thermal effect vaporization, and this leads to material removal. This represents a single step in the modeling. One material for which the evaporation laser cutting model can be applied is wax.

#### 4. FUSION LASER CUTTING

The erosion front consists of a zone of energetic initiation of the processes that propagate through the material at a given speed, depending on the energy input, the thermal properties of the material, its geometry, and the erosion mechanism. The erosion front appears as a layer of liquid due to the absorption of laser energy that is transformed into thermal energy. This layer is joined by the interface between the liquid layer and the solid material. The removal of material is due to the pressure difference that pulls it out of the kerf. The pressure difference is mainly due to the gas jet to which is added the pressure of evaporation or decomposition of the material.

For most for cut materials, there is both a molten and a vaporized form of the material. The energy source will be applied at the interface between the liquid and gaseous states, that is, at the surface of the melt. In some cases, the energy is absorbed by the gaseous medium and then partially reradiated to the solid or liquid. The radiation transmitted in the liquid or even solid material is absorbed.

Experimental data lead to liquid layer thickness of 0.3  $\mu\text{m}$  (in the case of metal cutting). Absorption occurs in a limited layer of 0.1  $\mu\text{m}$  in the molten material.

In the study of the propagation of the erosion front, the propagation of the solid-liquid interface is an isotherm at the melting temperature, Olsen [6].

The erosion front can be considered a semi-cylindrical surface. The cutting speed  $V_c$  is parallel to the surface of the piece and oriented in the cutting direction.

The  $V_M$  melting speed is normal to the surface of the erosion front.

It depends on the point considered. The angles between the direction of  $V_c$  and the surface normal in the horizontal and vertical planes are  $\beta$  and  $\alpha$ . The correlation between cutting speed and melting speed is given by:

$$V_m = V_c \cos\alpha \cos\beta \quad (9)$$

$V_m$  depends on the temperature gradient in the considered direction. The formula (9) shows that the melting speed is high in the central part and low towards the edges of the kerf. In the vertical plane, the  $V_M$  decreases with the curvature of the surface of the erosion front towards the bottom of the kerf. The melted layer has a half-moon shape. The thickness of the molten layer depends on the  $V_M$ . It undergoes variations that can be seen at the edges of the kerf as striations.

The energy transmitted by the liquid layer to the solid depends on the way the melt flows, more precisely if the flow is laminar or turbulent.

When the layer has a small thickness, the flow can be considered laminar and then the energy conducted in the melt is proportional to the temperature gradient in the melt. If the surface of the melt is below the boiling temperature, the forces that will remove the melt are gas pressure and surface tension. Neither of these will tend to decrease the thickness of the molten layer.

Gas pressure is constant or decreases along the erosion front, which is an additional cause for increasing the thickness of the liquid layer at the bottom of the kerf.

It is observed that evaporation, if it occurs, will occur in the area where the liquid layer has a greater thickness, i.e., towards the bottom of the kerf.

This model concludes that flow lines of molten material would cause striations.

The effective absorption of electromagnetic radiation and its guidance in the kerf are complex phenomena that are not considered by most authors.

A numerical evaluation of the temperature of the molten material layer depending on the parameters of the experiment and the absorptivity of the laser is presented in Schuöker and Abel [5]. The paper presents an analytical model of laser cutting assisted by a jet of reactive gas. The obtained results are evaluated numerically and applied to an experimental situation.

The objective functions pursued in the theoretical study are the determination of those quantities that are inaccessible to direct measurements, namely:

the temperature at the surface of the molten material layer, the influence of the cutting speed on the material removal, the contribution of melting and evaporation to the material removal.

The influencing factors considered are material constants (for steel), laser power, cut width, reaction energy, etc.

The main difficulty highlighted is the determination of the laser radiation attenuation in the waveguide represented by the cut and the polarization effect.

In modeling, these problems are easily solved by considering fractions of energy transformation, but the problems are found when establishing their values, especially stationary values.

Another problem raised by this type of model is the use of the shape and dimensions of the cut to appreciate the quantities of material involved and therefore the extent of the erosion process, although practically accessible, it is not the ideal one, having to be approximated, which leads to variations in the model.

The thickness of the liquid layer can be estimated (according to Olsen [6]) in the case of a laminar flow, the melt transmitting the necessary energy to the erosion front.

The maximum thickness of the molten material layer is given by:

$$t_{\max} = \frac{(T_S - T_M) K_M}{((T_S - T_M) C_P + H_M) w V_M} \quad [\text{m}] \quad (10)$$

$T_0$  - room temperature [K];  $T_M$  - melting temperature [K];  $T_S$  - melt surface temperature [K];  $K_M$  - thermal conductivity of the melt [W/m·K];  $C_P$  - heat capacity of metal [J/kg·K];  $H_M$  - heat of reaction [J];  $w$  - metal density [kg/m<sup>3</sup>];  $V_M$  - melting speed [m/s].

The minimum thickness of the molten material layer that can be obtained at the bottom of the kerf is obtained in a rough approximation considering the acceleration of the molten material by the pressure of the gas jet and neglecting the viscosity:

$$(t_{\min})^2 = \frac{w}{p} \cdot (V_c \cdot h)^2 \quad [\text{m}] \quad (11)$$

$p$  - pressure difference [Pa];  $h$  - material thickness [m];  $V_c$  - cutting speed [m/s]

Experimentally, the thickness of the erosion front can be highlighted by measuring the burrs that remain at the bottom of the kerf or in a stop cut. Another measurable quantity is the droplet ejection speed. This is:

$$V_E = \frac{h}{t_s} \cdot V_c \quad [\text{m/s}] \quad (12)$$

$t_s$  is the effective thickness of the liquid layer [m].

The study of the erosion front and the layer of melted material occupies an important place in

modeling because generally the phenomena that occur here are not accessible to direct experimental measurements.

## 5. CONCLUSIONS

Laser cutting has an important position in the field of metal cutting, mainly due to its ability to achieve high quality cuts at reasonable productivity. Mathematical modeling in laser cutting must answer the following problems after, Di Pietro [10]:

1. Ease of adjusting the process parameters and the flexibility of the process (for example the use of another material or different thicknesses of the same material).
2. Deviations of the cutting quality from the prescribed conditions are associated with drifts and slow disturbances of the cutting process such as: the variation of the properties of the piece, the fluctuations of the working gas, the power fluctuations and the spatial distribution of the radiation intensity, disturbances of the integrity of the optical devices used in the cutting system. All these impose the necessity of process monitoring.
3. To identify and compensate for errors that may occur, the correlation between the process and workpiece variables and the quality of the cut must be investigated. In this way, diagnosing the actual situation becomes extremely important, even if it is difficult to achieve it.

To know which parameters, need to be adjusted, how and when, modeling the process becomes essential and with it the need to investigate and direct the quality so that subsequent finishing operations of the cut surfaces are no longer necessary. The quality of a laser cut can be experienced by one or more criteria: macroscopic criteria (width of the cut, parallelism of the cut edges, the slope of the inner faces of the cut, slag adhesions) and microscopic criteria (the thermally affected area, surface roughness i.e. striations).

Most of the analytical models developed up to now have been related to the steady state of the cutting process.

Laser cutting is both dynamic and stochastic, with fluctuations in absorbed power, material composition and optical integrity, as well as other disturbances.

The models corresponding to the stationary stage were beneficial for obtaining the approximate order of magnitude of the different indicators, but they

have limited utility in the processes of processing management. For this, dynamic models must be developed that provide multiple resources for quality improvement.

Mathematical modeling in laser cutting must start from a concrete technological necessity and have finality in the optimization of manufacturing processes.

The present paper wants to bring back the attention on the elementary analytical modeling applied in laser cutting, an approach that is currently in decline due to the application of advanced simulation programs that do not give access to the basic aspects of the model.

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